DYNAMICS OF FALSE VACUUM BUBBLE IN VAIDYA SPACETIME

—Creation of a child universe in Vaidya spacetime with outgoing negative energy radiation—

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Abstract

This science report is based on a cooperative study with T. Mishima (Nihon University) and N. Yoshino (Tokyo Institute of Technology). The relativistic dynamics of a domain wall in a radiation fluid is studied. As one of applications, we demonstrate an interesting model in which a false vacuum bubble, in the case of a certain kind of surface stress-energy tensor, expands monotonically and at the same time can create a child universe.

1. Introduction

It is now generally accepted that the universe rapidly inflated after the big bang, as was proposed by K. Sato and A. Guth. By the gigantic spacetexpansion the initially hot region of the universe rapidly cools down to the temperature of the cosmological first order phase transition. Such regions would become the false-vacuum bubbles surrounding the true-vacuum bubbles (Fig. 1). We are supposed to be living now in one of the inflated regions of the universe, because the regions that did not inflate would have remained microscopic in size and would not accommodate life. In the inflationary universe model, the researches for the false-vacuum bubbles are considered to be very important and intriguing.

In 1981, K. Sato *et al.*[1] pointed out the possibility of the multipleproduction of universes by studying the motion of the false-vacuum bubble. After their works, several authors discussed whether the creation of a child universe occurs by producing a false vacuum bubble *in the laboratory,* that is, whether a false vacuum bubble prepared on a regular initial spacelike hypersurface can inflate permanently or not.[2)-4)] Here the regular initial spacelike hypersurface means the Cauchy surface on which the observer *in the laboratory* can manipulate the Cauchy data with no obstructions from some past singularities. In the classical theory, the above possibility is forbidden by a certain kind of no-go theorem[5] at least in spherical symmetric cases if the energy-momentum

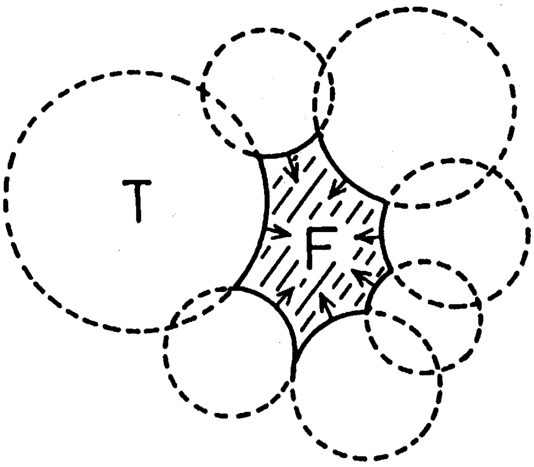

Fig. 1

tensor obeys the weak energy condition. Hence, other routes to escape from the no-go theorem are proposed in this paper. For example, the authors of[3),4)] pointed out the realization of the possibility by the use of quantum mechanical tunneling of the bubble itself.

2. Junction conditions

In this section, the junction conditions are derived from the Einstein equation. The junction conditions are the equation describing the dynamics of the domain wall between the true- and false-vacuum regions. The true-vacuum region is a region in which the energy density is zero and is stable both classically and quantum mechanically. On the other hand, the false-vacuum region is a region in which the energy density is non-zero and stable classically but unstable quantum mechanically.

The four-dimensional Einstein equation is

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi GT_{\mu\nu}, \tag{2.1}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, G is the Newton's gravitational constant, and $T_{\mu\nu}$ is the matter energy-momentum tensor.

Let us use a Gaussian normal coordinate system in the neighborhood

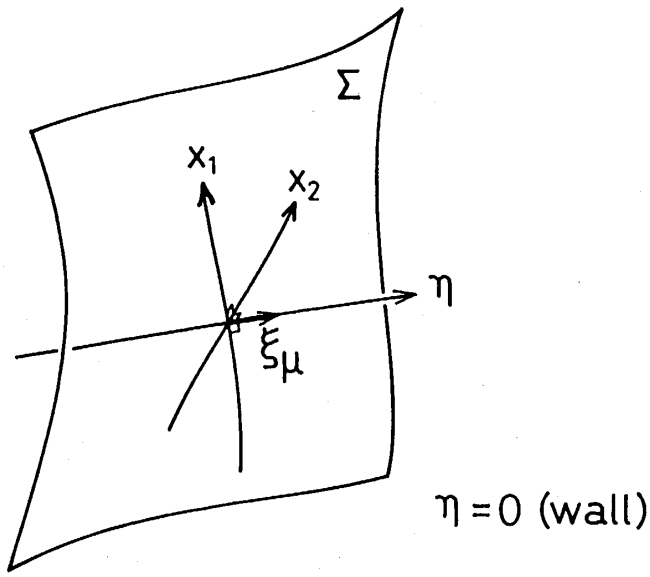

Fig. 2

of the wall for the simplicity. Let the (2+1)-dimensional spacetime hypersurface Σ be an evolutionary position of the domain wall (Fig. 2). The coordinate η is perpendicular to the hypersurface Σ as the distance in the poisitive direction from Σ. The equation $\eta=0$ indicates the hypersurface Σ. The coordinates $x^i (i=1, 2, 3)$ are the coordinates on Σ. The vector ξ_μ is a unit vector normal to Σ.

The extrinsic curvature K_{ij} corresponding to each η=const. hypersurface is a three-dimensional tensor expressed by

$$K_{ij}=\nabla_i \xi_j=-\Gamma^\eta_{ij}=-\frac{1}{2}\partial_\eta g_{ij} \tag{2.2}$$

where ∇_i is the four-dimensional covariant derivative, and g_{ij} is the metric of the hypersurface. The Einstein equation becomes

$$G^\eta_\eta=-\frac{1}{2}{}^{(3)}R+\frac{1}{2}[(TrK)^2-Tr(K^2)]=8\pi G T^\eta_\eta, \tag{2.3a}$$

$$G^\eta_i=D_m K^m_i-D_i(TrK)=8\pi G T^\eta_i, \tag{2.3b}$$

$$\begin{aligned} G^i_j={}^{(3)}G^i_j-\partial_\eta(K^i_j-\delta^i_j TrK)-(TrK)K^i_j \\ +\frac{1}{2}\delta^i_j[TrK^2+(TrK)^2]=8\pi G T^i_j, \end{aligned} \tag{2.3c}$$

where ∂ is the partial derivative and D the three-dimensional covariant derivative with respect to the metric g_{ij} defined on Σ.

The energy-momentum tensor $T^{\mu\nu}$ is expected o have a δ-function singularity at the domain wall and is given by

$$T^{\mu\nu}(x)=S^{\mu\nu}(x^i)\delta(\eta)+\Theta(-\eta)T^{\mu\nu}_-(x)+\Theta(\eta)T^{\mu\nu}_+(x), \tag{2.4}$$

where $S^{\mu\nu}$ is the surface stress-energy tensor and $-$ is inside region of the domain wall and $+$ is outside region of the domain wall. Throughout this article we will use the thin wall approximation assuming that the thickness of the domain wall is negligible because the scalar field Φ, which describes the domain wall configuration, changes sharply at the domain wall.

Inserting the energy-momentum tensor of Eq. (2.4) into the field equation (2.3), one sees that (2.3a) and (2.3b) are satisfied automatically if the continuous solution g_{ij} is given in the neibourhood of $\eta=0$. The equation (2.3c) is the only non-trivial condition and $lim\int^{\epsilon}_{\epsilon} d\eta$x (2.3c) leads to the junction condition

$$\lim[K^i_j(\eta=+\epsilon)-K^i_j(\eta=-\epsilon)]=-8\pi G\left(S^i_j-\frac{1}{2}\delta^i_j TrS\right).$$

3. Surface stress energy of a domain wall

The energy-momentum conservation law restricts the surface stress-energy defined by Eq. (2.4) to a certain extent. The equations for renergy-momemtum conservation in the Gaussian normal coordinates read

$$\nabla \nu T^{i}\nu = D_j T^{i}j + \partial_\eta T^{i}\eta + 2K^i_j T^j\eta + (TrK)T^i\eta = 0\,, \tag{3.1a}$$
$$\nabla_\nu T^{\eta\nu} = D_i T^{\eta i} + \partial_\eta T^{\mu\mu} - K_{ij}T^{ij} + (TrK)T^{\eta\eta} = 0\,. \tag{3.1b}$$

Inserting Eq. (2.4) into Eq. (3.1a), one finds

$$\begin{aligned}\nabla_\nu T^{i\nu} &= S^{i\eta}\delta(\eta) \\ &\quad + [D_j S^{ij} - T^{i\eta}_- + T^{i\eta}_+ + 2K^i_j S^{j\eta} + KS^{i\eta}]\delta(\eta) \\ &\quad + [D_j T^{ji}_- + \partial_\eta T^{i\eta}_- + 2K^i_j T^{j\eta}_- + KT^{i\eta}_-]\Theta(-\eta) \\ &\quad + [D_j T^{ji}_+ + \partial_\eta T^{i\eta}_+ + 2K^i_j T^{j\eta}_+ + KT^{i\eta}_+]\Theta(-\eta) \\ &= 0,\end{aligned} \tag{3.2}$$

where the prime denotes differentiation with respect to η. The consistency of Eq. (3.2) at $\eta=0$ demands

$$S^{i\eta} = 0\,, \tag{3.3}$$
$$D_j S^{ji} = T^{i\eta}_- - T^{i\eta}_+\,. \tag{3.4}$$

Inserting Eq. (2.4) into Eg. (3.1b), one can get

$$S^{\eta\eta} = 0, \tag{3.5}$$
$$\overline{K}_{ij} S^{ij} = T^{\eta\eta}_+ - T^{\eta\eta}_-. \tag{3.6}$$

Combining the orthogonality conditions (3.3) and (3.5) with rotational invariance, one concludes that $S^{\mu\nu}$ can be written as

$$S^{\mu\nu} = \sigma(\tau)U^\mu U^\nu - \zeta(\tau)(h^{\mu\nu} + U^\mu U^\nu)\,, \tag{3.7}$$

where

$$h^{\mu\nu} = g^{\mu\nu} - \xi^\mu \xi^\nu \tag{3.8}$$

is the metric projected into the hypersurface of the wall, and

$$U\mu = (1,0,0,0)\,, \tag{3.9}$$

is the four-velocity of the domain wall. Here σ is the surface density of the domain wall, and ζ is the surface tension. Rotational invariance also implies that the metric on the domain wall can be written as

$$ds^2 = -d\tau^2 + r(\tau)^2 d\Omega^2\,, \tag{3.10}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\o^2$ and τ is the proper time of the wall. Calculating Eq. (3.7), we have

$$S^{\tau\tau} = \sigma(\tau)\,, \tag{3.11a}$$
$$S^{\theta\theta} = -\frac{\zeta(\tau)}{r^2}\,, \tag{3.11b}$$
$$S^{\o\o} = -\frac{\zeta(\tau)}{r^2\sin^2\theta}\,, \tag{3.11c}$$
$$\text{others} = 0. \tag{3.11d}$$

By Eq. (3.11), we can get

$$D_j S^{\tau j} = \dot{\sigma}(\tau) + 2\frac{\dot{r}}{r}(\sigma(\tau) - \zeta(\tau))\,, \tag{3.12a}$$
$$D_j S^{\theta j} = 0\,, \tag{3.12b}$$
$$D_j S^{\o j} = 0\,, \tag{3.12c}$$

where the overdot denotes a derivative with respect to τ. From Eq. (3.4) and Eq. (3.12a), we can deduce

$$\dot{\sigma}(\tau)+2\frac{\dot{r}}{r}(\sigma(\tau)-\zeta(\tau))=T_{-}^{\tau\eta}-T_{+}^{\tau\eta}. \tag{3.13}$$

4. Equation of motion for a domain wall

In the inside region the energy-momentum tensor $T_{-}^{\mu\nu}$ is dominated by nonzero vacuum energy, $-(3\chi^2/8\pi G)g_{\mu\nu}$. The inside of the wall is described as the region of the de Sitter space,

$$ds^2=-(1-\chi^2r^2)dt^2+(1-\chi^2r^2)^{-1}dr^2+r^2d\Omega^2. \tag{4.1}$$

Outside the domain wall, due to the existence of the null energy flux radiated by the wall, the energy-momentum tensor $T_{+}^{\mu\nu}$ can be written as

$$T_{+}^{\mu\nu}=\mp\frac{\partial_v \mathrm{m}(v)}{4\pi r^2}l^{\eta}l^{\nu}, \tag{4.2}$$

where $\mp$ sings denote in-going or out-going respectively. Then the outside geometry is given by Vaidya's radiating metric[7]

$$ds^2=-\left(1-\frac{2Gm(v)}{r}\right)dv^2-2dvdr+r^2d\Omega^2, \tag{4.3}$$

where l^{μ} is radial null vector and v is called retarted time coordinate (in fact, v coincides with the observer's time at infinity). The function $m(v)$ is the Bondi mass, and is determined by the basic equations below.

Following Israel,[6] the most important equation of motion for the domain wall is given as a junction condition deduced from the Einstein equation (2.1). By evaluating Eq. (2.5), the junction condition reads

$$K_j^{i+}-K_j^{i-}=-4\pi G\sigma, \tag{4.4}$$

where the superscripts $+$ and $-$ denote the outside and the inside respectively.

We will evaluate the $\theta\theta$ component of Eq. (4.4). We begin by calculating the normal vector as seen by a Vaidya observer. Because the domain wall is spherically symmetric, the four-velocity of any point on the domain wall assumes the form

$$U_V^{\mu}=\left(N,N\frac{dr}{dv},0,0\right), \tag{4.5a}$$

$$U_{V\mu}=\left(\left(-a\pm\frac{dr}{dv}\right)N,\pm \mathrm{N},0,0\right), \tag{4.5b}$$

in the Vaidya coordinates, where $N=(1-2Gm/r-2dr/dv)^{-1/2}$. Because the unit normal ξ_{μ} is orthogonal to U^{μ}, we have ξ_{μ} and ξ^{μ};

$$\xi_{\mu}=\left(-N\frac{dr}{dv},N,0,0\right), \tag{4.6a}$$

$$\xi^{\mu}=\left(\pm N,N\left(1-\frac{2m}{r}\mp\frac{dr}{dv}\right),0,0\right), \tag{4.6b}$$

where the upper and lower signs denote in-going and out-going respectively. By applying Eq. (2.2) to our Vaidya system, we have

$$K^{+}_{\theta\theta}(Vaidya)=\frac{1}{2}\partial_{\eta}g_{\theta\theta}$$
$$=\frac{1}{2}\zeta^{\mu}\partial_{\mu}r^2$$
$$=N\left(1-\frac{2Gm}{r}\mp\frac{dr}{\mathrm{dv}}\right)r$$
$$=\gamma\beta_V, \tag{4.7}$$

where $\beta_V=\pm(\dot{r}^2+1-2Gm/r)^{1/2}$. For the last equality we have used

$$\frac{d\gamma}{dv}=\dot{r}\left(+\dot{r}+\left(\dot{r}^2+1-\frac{2Gm}{r}\right)^{1/2}\right),$$

and

$$\frac{dr}{dv}=\mp\dot{r}+\left(\dot{r}^2+1-\frac{2Gm}{r}\right)^{1/2}=\left(1-\frac{2Gm}{r}-2\frac{dr}{dv}\right)^{1/2},$$

where$\pm$signs of first term denote in-going and out-going. By the same method,[2] we get

$$K^{-}_{\theta\theta}(\text{de Sitter})=r\beta_D, \tag{4.8}$$

where $\beta_D=\pm(\dot{r}^2+1-\chi^2r^2)^{1/2}$.

From Eq. (4.7) and (4.8), the $\theta\theta$ component of Eq. (4.4) is written as

$$\beta_v-\beta_D=-4\pi G\sigma(\tau)r(\tau). \tag{4.9}$$

The junction condition (4.9) in the case of the wall with radiation gives the same equation as the junction condition in the case of the wall without radiation except for the valiable $m(v)$. The $\tau\tau$ component $K_{\tau\tau}$ is given by

$$K_{\tau\tau}=\xi_{\tau;\tau}=U^{\mu}U^{\nu}\xi_{\mu;\nu}=-\xi_{\mu}U^{\nu}U^{\mu}_{;\nu}=-\xi_{\mu}\frac{DU^{\mu}}{D_{\tau}}, \tag{4.10}$$

where $DU^{\mu}/D_{\tau}=dU^{\mu}/d_{\tau}+\Gamma^{\mu}_{\lambda\sigma}U^{\lambda}U^{\sigma}$ is the covariant acceleration of the wall. $K_{\tau\tau}$ is the component of the covariant acceleration in the normal direction. By applying Eq. (4.10) to the Vaidya system, we obtain

$$K^{-}_{\tau\tau}(\text{Vaidya})=-N^2\frac{d^2r}{d\tau dv}+\frac{\partial_{\gamma}\left(1-\dfrac{2Gm}{r}\right)}{2}$$
$$\times N^3\left(2\frac{dr}{dv}-\left(1-\frac{2Gm}{r}\right)\right)+\frac{\partial_v\alpha}{2}N^3, \tag{4.11}$$

where $N=(1-2Gm/r-2dr/dv)^{-1/2}$. By the same method[2], we get

$$K^{+}_{\tau\tau}(\text{de Sitter})=\frac{(r-\chi^2r)}{(1-\chi^2r+\dot{r}^2)^{1/2}}. \tag{4.12}$$

From Eqs. (4.11) and (4.12), the $\tau\tau$ component of Eq. (4.4) is written as

$$N^2\frac{d^2r}{d\tau dv}-\frac{\partial_{\gamma}\alpha}{2}N^3\left(2\frac{dr}{dv}-\left(1-\frac{2Gm}{r}\right)\right)$$
$$-\frac{\partial_v\alpha}{2}N^3-\frac{r-\chi^2r}{(1-\chi^2r+\dot{r}^2)^{1/2}}$$

$$=4\pi G(\sigma-2\zeta)\,, \tag{4.13}$$

where $N=(1-2Gm/r-2dr/dv)^{-1/2}$.

We can easily see that Eq. (4.13) is simply the proper-time derivative of Eq. (4.9). Indeed, this case is the same as the case of the wall without radiation.

By assuming that the energy-momentum tensor $T_{\mu\nu}$ of the de Sitter space is very small, we can write as

$$T^{-}_{\mu\nu}=0\,. \tag{4.14}$$

The null vector l^{μ} in Vaidya's radiating schwarzschild metric is given as

$$l^{\mu}=\frac{1}{\left(1-\dfrac{2Gm(v)}{r}-2\dfrac{dr}{dv}\right)^{1/2}}(1,\mp1,0,0)\,, \tag{4.15}$$

where the upper sign $-$ and the lower sign $+$ denote in-going and out-going, respectively, and each component denotes τ, η, θ, $ø$ component. From Eqs. (4.2), (4.14) and (4.15), the right hand of Eq. (3.12) can be gotten

$$\pm\frac{\partial_v m(v)}{4\pi r^2}\,. \tag{4.16}$$

From Eqs. (3.12), (4.14) and (4.16), we can get

$$\frac{d(4\pi r^2\sigma)}{d\tau}-\zeta\frac{d(4\pi r^2)}{d\tau}=\frac{\partial_\tau m}{(\mp\dot{r}+\beta_V)}\,. \tag{4.17}$$

where the upper sign $-$ and the lower sign $+$ denote in-going and out-going respectively and the above equation represents the energy-momentum conservation law. The right hand side corresponds to the energy flow which the domain wall have for the in-going or out-going radiation.

The motion for a domain wall is determined by the two independent equations (4.9) and (4.17) and another additional condition, which we will explain in the next chapter. Let us recall that the equation (4.9) is given by the junction condition and Eq. (4.17) comes from the energy conservation law.

5. Solution of the equations of motion

After a simple calculation from Eqs. (4.4), (4.7) and (4.8), we can see that the junction condition is written by

$$\dot{r}+V(r,m,\sigma)=-1,$$
$$V\equiv-\chi^2-\left[\frac{2Gm+((4\pi G\sigma)^2-\chi^2)r^3}{8\pi G\sigma r^2}\right]^2. \tag{5.1}$$

This equation has the same form as the Schwarzschild vacuum case derived by Blau *et al.*[2], although m and σ depend on the proper time τ in our present case.

For the previously mentioned additional condition, the surface-

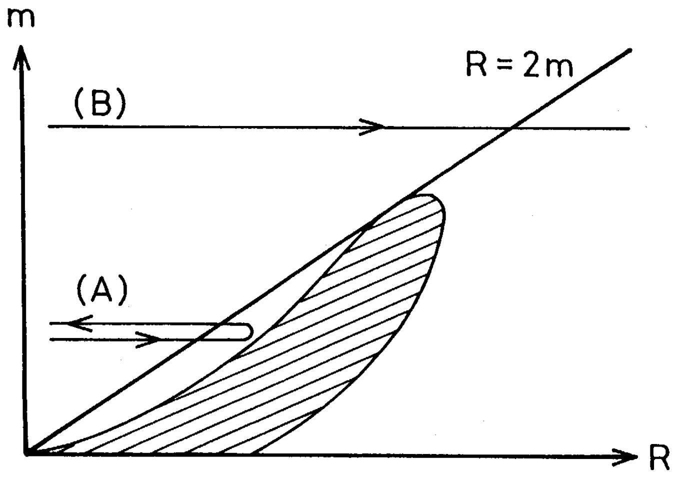

Fig. 3

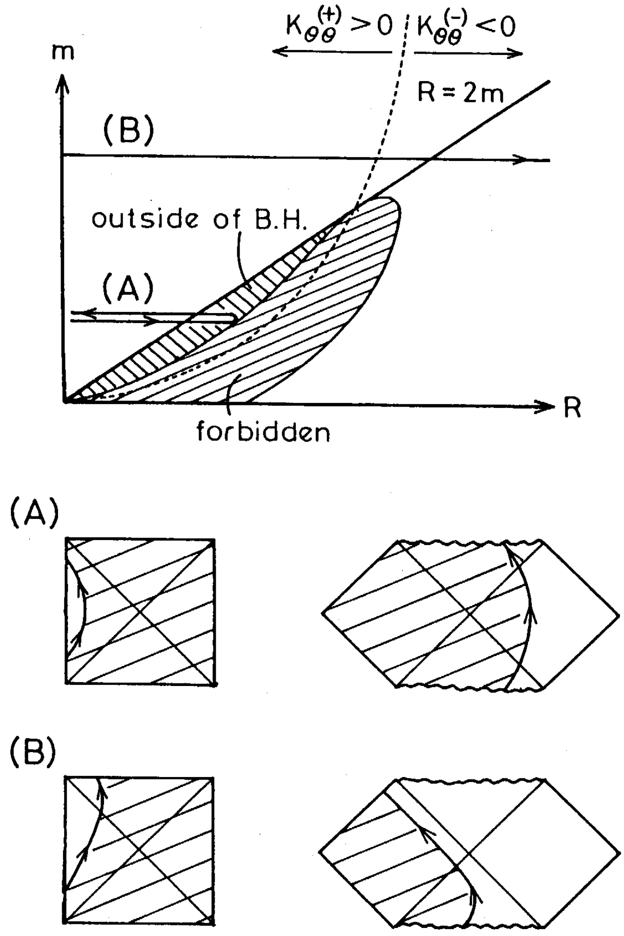

Fig. 4

energy tensor approximates to $S^i_j=\sigma\delta^i_j$ where $\dot{\sigma}=0$.[2)] That is, $\sigma=\zeta=$ *constant* (called as normal vacuum case in this talk). From this result, the Eq. (4.17) reads $m=const.$ and the problem is reduced to the one-

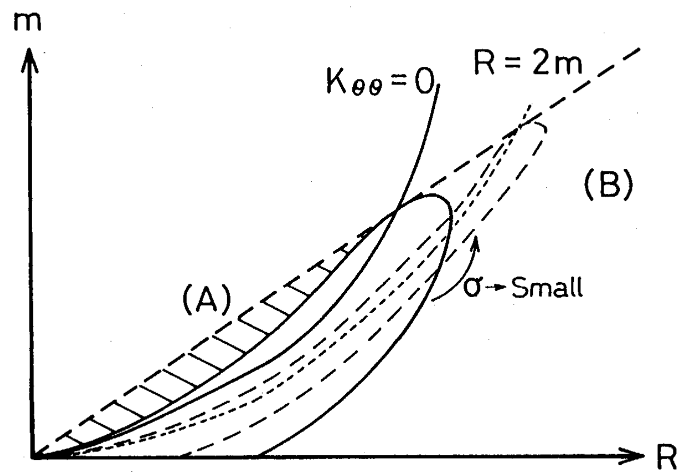

Fig. 5

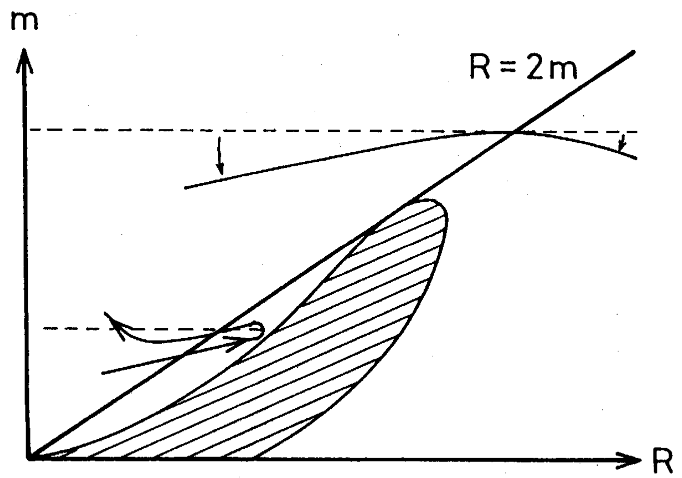

Fig. 6

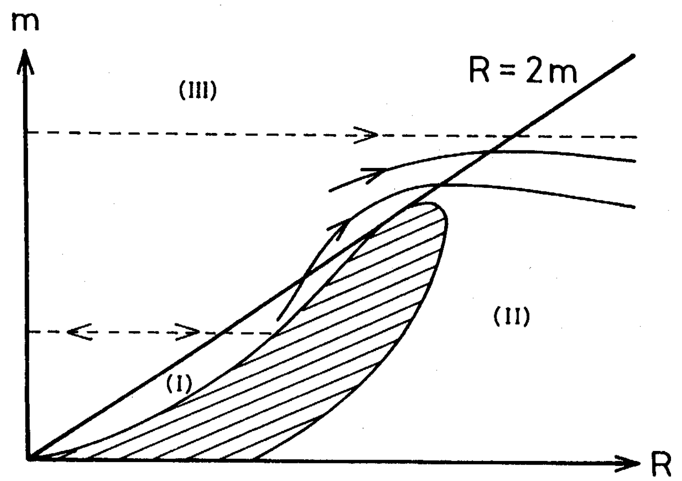

Fig. 7

dimensional potential problem based on the Eq. (5.10). The trajectories of the wall are described as horizontal dashed lines in $(r-m)$ diagram (Fig. 3) and are classified into several types.[2] The diagrams show the behavior in each of the de Sitter and the Vaidya space-time. When the value of σ is bigger, the shape of the potential becomes narrow to right side (Fig. 5). In the following, for the additional condition, we generalize the above case to $\sigma \neq \zeta$ case. In contrast with the normal vacuum case, the Eq. (5.1) becomes the non-trivial form,

$$\frac{dm}{dr} = 2\Delta r \left(\frac{dv}{d\tau}\right)^{-1}, \tag{5.2}$$

where the Planck units are introduced ($l_p m \to m$, $l_p^{-1} r \to r$, $4\pi l_p^3 \sigma \to \sigma, \cdots$), and $\Delta = \sigma - \zeta \neq 0$. We will restrict in the case of out-going negative flux, although this 0, equation is consistent with both in-going flux and out-going flux. From the examination of this equation, one can find that the wall trajectories are modified like the real lines shown in Fig. 6 if $\Delta > 0$. Then in the case of a large enough Δ, as easily expected, the wall can move from the region of *the laboratory* side (I) to the other side II) with its proper radius expanding monotonously, avoiding the potential barrier (the shaded region in Fig. 7). One can also confirm the existence of such solutions by numerical evaluation, and find the possibility of the creation of a child universe. For further understanding of the global structure of the spacetime, the conformal diagram of the outside space-time is shown in Fig. 8 whose apparent horizons are described as the line (a) and the line (b). First of all, it should be noted that the former apparent horizon (a) is always null and coincides with the future event horizon itself, while the latter one (b) becomes time-like during the

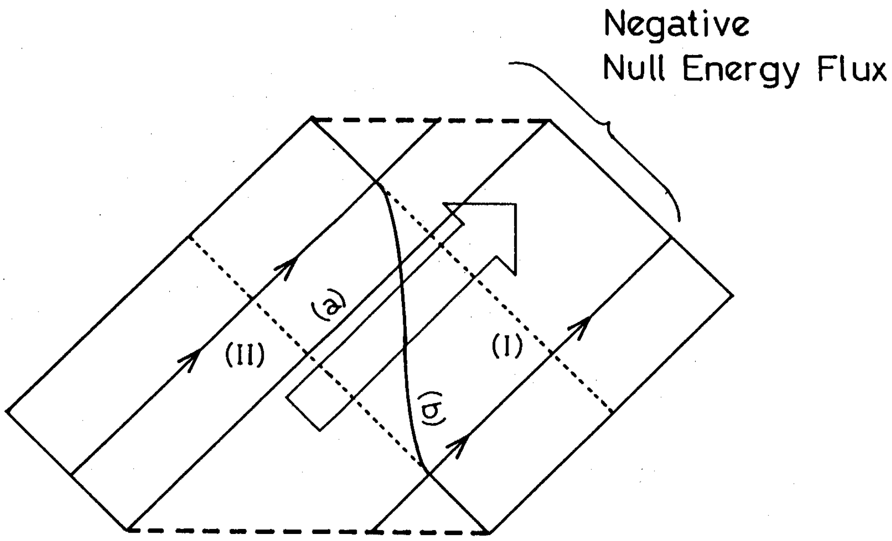

Fig. 8

existence of out-going negative energy null flux and does not coincide with the past event horizon. From this diagram one can easily understand how the wall moves in the other side from *the laboratory* side. From Eq. (5.1), the relation of r and m is plotted (Fig. 3). The forbidden region is indicated by the oblique-line region. Both the allowed region and the outside region of Black Hole are indicated by shading.

6. Conclusion

We have presented another possible scenario of the creation of a child universe. Our strategy adopted here is to break the weak energy condition by introducing the domain wall which radiates negative energy flux. The formalism applied to the wall whose surface stress-energy tensor is different from the normal vacuum one. The difference is characterized by $\Delta=\sigma-\zeta$. We showed that the gate for *the new world* is opened if Δ is large enough. As compared with the way by quantum tunneling, the treatment discussed here presents the real route to *the new world* in the real spacetime. However, the surface stress-energy tensor of the wall must be well-devised (perhaps quantum mechanically). As the conclusions from this study is still not conclusive, and futher study should be conducted.

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